

Experimental investigations with respect to the applicability of the Bohm criterion

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Abstract

The streaming of a plasma in a magnetic nozzle field in front of a terminating target is studied by means of laser induced fluorescence. In contrast to expectations basing on Bohm's criterion it is found that the flow is generally sub-sonic in the region behind the last magnetic coil. Half-sided Maxwellian ion-distribution functions with Mach numbers around 0.5 are typical for standard recycling conditions within a distance of the mean free path length from the target. Larger flow velocities, possibly exceeding $M = 1$, are however observed under maximum gas pumping conditions. The importance of the recycling neutrals with respect to the streaming behavior is also corroborated by numerical simulations.

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1. Introduction

Bohm's criterion includes an important statement on the flow behavior of a plasma in contact with a material surface. It postulates that the Mach number ($M = \text{streaming velocity/sound velocity}$) should become at least unity at the edge of the electrostatic sheath, i.e. a few Debye lengths ahead of the surface (apart from the original paper by Bohm [1] the reader is referred to Refs. [2,3] for the derivation of the criterion). In order to satisfy this condition the ions have to be accelerated in the main part of the plasma. This may happen in an adjacent 'pre-sheath' by an electric field or remote from

the edge in the source region or elsewhere either by E-fields or by the pressure gradient.

The criterion is specially applied as a boundary condition to interpret Langmuir probe data ($M = 1$) or to simulate the scrape-off layer plasma streaming onto the divertor target plates in fusion devices. As will be shown in Section 2 there are in particular essential changes to be expected – leading to $M \gg 1$ – when the plasma is streaming in a magnetic field of decreasing strength (magnetic nozzle). The crucial influence of a corrugated magnetic field on the streaming behavior became also obvious from B2–Eirene code calculations [4]. The applied code contains the full set of equations and boundary conditions as used in up to date tokamak divertor simulations but a different grid structure to comply with the different geometry. It predicts supersonic plasma flow (with Mach numbers up to about 2) for the last 30 cm ahead of the neutralizer plate in case of typical

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hydrogen discharges (unfortunately there is no argon atomic data package available for the Eirene Monte Carlo part).

First measurements carried out in the plasma generator PSI-2 arose questions whether the plasma flow is actually becoming sonic or super sonic in front of the neutralizer plate [5]. In these experiments we studied the change of flow induced by a magnetic nozzle using Mach probes. In this paper we report on improved measurements using Laser Induced Fluorescence (LIF).

1.1. The plasma generator PSI-2

Fig. 1 shows the Plasma Generator PSI-2. A steady plasma (argon in these experiments) is produced in the cathode–anode region which, confined by a magnetic field, drifts for about two meters in the vacuum chamber until it is terminated at the neutralizer plate at the end of the device. Ion temperatures around $T_i = 1\text{--}5$ eV at densities of about $n_i = 10^{18} \text{ m}^{-3}$ are typical plasma parameters. The PSI-2 chamber is surrounded by six coils providing an axial magnetic field of $B = 0.1\text{--}0.25$ T. Due to the small number of coils there are noticeable inhomogeneities leading to more or less pronounced magnetic mirror configurations which are important with respect to plasma flow. The magnetic configuration is defined by the flux surfaces $\psi(r, z) = \text{const}$. Close to the axis these are given by $\psi = B(z)r^2/2$ where $B(z)$ is the field on axis. The variation of the plasma radius ($a \approx 3$ cm) according to $\psi = \text{const}$. could be verified by optical measurements.

1.2. Laser induced fluorescence

For the experimental investigation of the plasma flow a narrow bandwidth tunable laser system was used. The laser beam could be launched alternating from both directions along the axis into the device (see Fig. 1). The wavelength of the laser light is scanned around $\lambda = 611.492$ nm in order to excite the Ar^+ ions from the metastable level $3d' \text{ G}_{9/2}$ to level $4p' \text{ }^2\text{F}_{7/2}$. Using

photomultipliers the fluorescence light produced in the spontaneous transition to $4s' \text{ }^2\text{D}_{5/2}$ was observed at two different positions: (1) in the main plasma 0.75 m away from neutralizer plate and (2) close to the plate in about 5 cm distance. As the bandwidth of the laser is much smaller than the Doppler broadened lines the laser excites only particles within a small range ($v \dots v + \Delta v$) of the ion velocity distribution (ivdf). Recording the fluorescence intensity during a wavelength scan gives us the projection of the ivdf parallel to the magnetic field. This function contains all kind of information; it allows in particular to determine the streaming velocity and the ion temperature.

2. Fluid model description of the plasma

We apply the equations of continuity and momentum for electrons and ions. Instead of the energy equation we assume adiabatic ($\gamma_i = 5/3$) or isothermal ($\gamma_e = 1$) processes yielding the relations $p_j = C_j n_j^{\gamma_j}$ in addition to the equations of state $p_j = n_j T_j$, with $j = i, e$. We consider the 1D flow along the z -axis and assume quasi-neutrality ($n_e = n_i = n$). Since the plasma region behind the anode is free of current (floating target) both species have equal flow velocities $\mathbf{u}_e = \mathbf{u}_i = u\mathbf{B}/B$. The continuity and momentum equations then take the form

$$B \frac{d}{ds} \left(\frac{un}{B} \right) = n v_{\text{ion}} \quad (1)$$

and

$$\frac{d}{ds} \left(m_i \frac{u^2}{2} + \frac{\gamma_e T_e}{\gamma_e - 1} + \frac{\gamma_i T_i}{\gamma_i - 1} \right) = -v_{\text{tot}} m_i u, \quad (2)$$

where $s \approx z$ is the length of arc along the field lines. Eq. (2) is obtained by adding the equations for electrons and ions and neglecting the electron inertia. The electric field and the Coulomb friction forces are thus eliminated. Here the total collision frequency $v_{\text{tot}} = v_{\text{ion}} + v_{\text{cx}} + v_{\text{el}}$ contains in addition to ionization also charge exchange and elastic collisions between ions and neutrals.

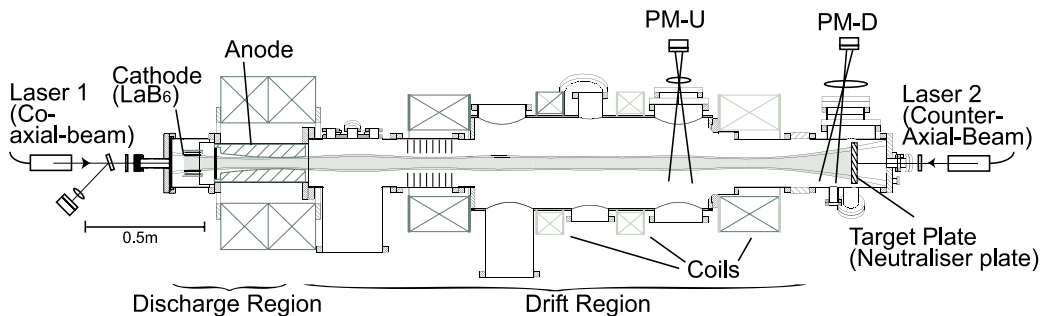


Fig. 1. LIF Measurements at PSI-2.

2.1. Source-free case – the magnetic Laval Nozzle

Let us assume that the particle sources and the collisional momentum losses can be neglected as it is usually the case for a normal gas. Then Eqs. (1) and (2) define two invariants, the quantity (un/B) and the total enthalpy (the expression in the bracket in Eq. (2)). If the flux density $\Gamma = nu$ is independent of radius (the gas case, which could be shown experimentally to hold also for our plasmas) the first invariant becomes equivalent with the total particle flux $\Phi = \Gamma A$ since the cross-section is given by $A(z) = \pi r^2 = 2\psi B^{-1}(z)$. Rather simple expressions are obtained for the special case $\gamma_e = \gamma_i = \gamma$ (qualitatively equal results are obtained for $\gamma_e \neq \gamma_i$ from the numerical solutions of Eqs. (4) and (5) when the sources are omitted). Then, after introducing the ion sound velocity $c = [\gamma(T_e + T_i)/m_i]^{1/2}$ and the Mach number $M = u/c$, the flux density becomes a function of M only

$$\Gamma \propto M \left[1 + (\gamma - 1)M^2/2 \right]^{-\frac{\gamma+1}{2(\gamma-1)}}. \quad (3)$$

From this expression the limiting case $\gamma = 1$ is obtained to $\Gamma \propto M \exp(-M^2/2)$ using $\lim_{x \rightarrow 0} (1+x)^{1/x} = e$. Most important is the fact that – for any $\gamma \geq 1$ – Γ reaches maximum at $M = 1$. Furthermore, due to conservation of Γ/B it can be inferred that whenever $M \geq 1$ will be attained, there will be $M = 1$ at the location of maximum B , i.e. the narrowest cross-section. In case of a gas the pressure ratios at the entrance and exit of the Laval Nozzle determine whether the flow will stay subsonic or pass into the supersonic regime $M > 1$. In contrast, in a plasma the Bohm criterion tells us that we have always to postulate $M \geq 1$ at the sheath edge of the neutralizer plate. These two requirements can only be satisfied if the flow is supersonic and monotonously increasing behind the position of maximum B (last coil) and the neutralizer plate. The Mach number at the target M_T can then be calculated from the relation $\Gamma(M_T)/B_T = \Gamma(1)/B_{\max}$. For the actual ratio $B_T/B_{\max} = 0.25$ we obtain $M_T = 2.34$ to 3.44 assuming $\gamma = 1$ and $5/3$, respectively.

2.2. Plasma flow under realistic conditions

Ignoring the particle sources is in fact not well justified under the actual plasma conditions. Although attempts were made to reduce these sources as far as possible by enhancing the pumping of neutrals, there remains the recycling of the atoms being released from the target and which become re-ionized during the first passage through the plasma at the end section. Their influence can be estimated from the flux balance $\phi_i = -\phi_n$, where ϕ_n denotes the total flux of the recycling neutrals which leave the plate with a relatively small velocity. The

probability for ionization during the first passage is about 60%. The ratio of the primary ions to those produced from the recycling neutrals is then estimated to 37%.

Including the sources Eqs. (1) and (2) can be transformed to

$$u' = \frac{M^2 v_{\text{tot}} + M c B' / B + v_{\text{ion}}}{1 - M^2}, \quad (4)$$

$$\frac{n'}{n} = - \frac{M^2 B' / B + M (v_{\text{tot}} + v_{\text{ion}}) / c}{1 - M^2} \quad (5)$$

with $\partial/\partial s = '$ and $T'_i/T_i = (\gamma_i - 1)n'/n$. Note that the singularity at $M = 1$ in the above equations causes a change of sign when passing the sonic border. The equations given above have been numerically solved taking the sources into account as they are obtained from Monte-Carlo simulations. Starting at the neutralizer plate with $M = 0.99$ and calculating backwards in the negative direction it is possible to stay within the subsonic range. It is found that the sources result in steep gradients of all plasma parameters within the last few centimeters. Values of $M \approx 0.5$, as measured about 5 cm ahead of the plate, are thus achievable – although the numerical results depend very sensitively on the details of the source distribution.

3. Measured ion velocity distributions

The measured ivdf for two different discharge regimes *A* ($p_n = 4.3 \times 10^{-4}$ mbar) and *B* ($p_n = 1.4 \times 10^{-4}$ mbar) are shown in Figs. 2 and 3. Note that the diameters of the co-axial (7 cm) and counter-axial (1.5 cm) beams near the target plate differ significantly. The signal-to-noise ratio is hence much better in the first case while the spatial resolution is reduced compared to the counter beam. Fig. 2 shows the typical results. Seventy-five centimeters away from the target the ivdf is almost a perfect Maxwellian, showing a small shift of about $M \approx 0.1$. In contrast, closer to target ($\Delta z = 5$ cm, bottom picture) the functions become approximately half-sided Maxwellians. This is particularly well resolved by the counter-axial beam which is confined to the near axis region. The Mach number is considerably enhanced compared to the more distant case but stays clearly in the subsonic range around $M = 0.5$. The approach to half-sided Maxwellian distributions is also predicted by theory [6,7]. It can be understood by noticing that for distances (Δz) smaller than the ion mean free path length (λ_i) the particles moving in negative direction are lacking because the neutralizer plate does not reflect them. For the experimental conditions it is estimated $\lambda_i \approx \Delta z = 5$ cm. Much more complicated distributions are found under low recycling

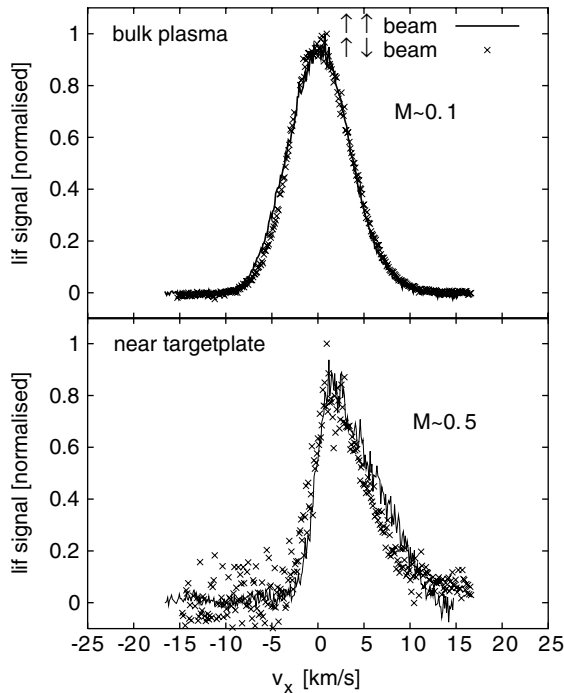


Fig. 2. LIF measurements of the ion distribution function (case A).

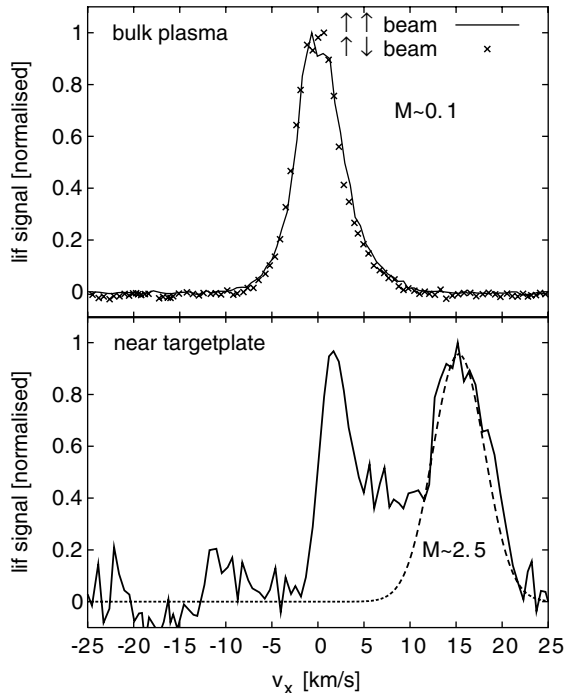


Fig. 3. As Fig. 2 but for lower recycling conditions (case B). In the bottom part only the data for the counter beam are presented. A shifted Maxwellian with $M = 2.5$ has been inserted for comparison.

Table 1

Evaluation of measurements for the cases (A) and (B)

Case	Position	u [m/s]	T_i [eV]	T_e [eV]	M	M_{fit}
A	Bulk	330	4.0	7	0.06	≈ 0
A	Target	2900	2.5	8	0.56	–
B	Bulk	180	2.6	9	0.03	0.09
B	Target	9100	17.7	9	1.01	–

The electron temperatures were obtained from Langmuir probe measurements.

conditions, case (B) shown in Fig. 3. Apart from extended tails there are structures not easily to explain. We suppose that part of the structure is to be attributed to the cold ions being produced by ionization of the recycling neutrals.

For the evaluation of the streaming velocity and T_i two different methods are used: (1) fitting the data to a Maxwellian distribution, and (2) calculating the first and second velocity moments by integration over the measured ivdf. Since the distributions obtained near the neutralizer plate show strong deviations from a Maxwellian, the fitting method is not suitable in these cases. Some results are presented in Table 1.

4. Discussion

It is found that in a distance of 75 cm from the target plate the ion distribution function is close to a Maxwellian but with a small asymmetry leading to Mach numbers of $M \approx 0.1$. Close to the target surface the distribution function approaches a half-sided Maxwellian with $M \approx 0.5$. The streaming velocity is hence significantly lower than $M \geq 2$ predicted by theory (without sources) for the actual magnetic arrangement and position of measurement. Attempts to enhance the flow velocity were finally successful by reducing the neutral gas pressure in front of the target by means of enforced pumping. In this case the deviations from a Maxwellian become even more pronounced and a temperature cannot be assigned anymore. Depending on interpretation Mach numbers of about 1 and larger are derived from the measurements in this case.

In conclusion, Bohm's criterion is likely to be satisfied always in its strict form, i.e. $M \geq 1$ at the edge of the sheath. However, its practical use as a boundary condition may be constrained by recycling that can result in a dramatic reduction of the flow velocity within a distance of about the mean free path length of the neutrals. Such problems are particularly to be expected when the plasma in total is in contact with the target as it was in these investigations. For small objects, like probes, on the other hand, the recycling problems should be relaxed for geometrical reasons. Hence, for

small probes in a strongly decaying magnetic field the Bohm criterion should take the form $M \gg 1$ which might be possible to substantiate experimentally.

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